1. **NSIDC data format? Slide 8**

They are stored in netCDF (.nc) format. There is convenient python package (package name netCDF4) to retrieve data from these files.

NSIDC provides python code to download files over a large period of time in single batch.

1. **What’s ferromagnetic? What’s paramagnetic? Slide 12 & 13**

**Ferromagnetism** is a property of certain materials (such as iron, nickel, cobalt) that results in a significant, observable [magnetic permeability](https://en.wikipedia.org/wiki/Magnetic_permeability), allowing the material to form a [permanent magnet](https://en.wikipedia.org/wiki/Permanent_magnet). Ferromagnetic materials are familiar metals that are noticeably attracted to a magnet, a consequence of their substantial magnetic permeability.

**Paramagnetism** is a form of [magnetism](https://en.wikipedia.org/wiki/Magnetism) whereby some materials are only weakly attracted by an externally applied [magnetic field](https://en.wikipedia.org/wiki/Magnetic_field).

**In microscope or statistical physics, Ferromagnetism** means that the spin (magnetic orientation) of the atoms are strongly aligned**. Paramagnetism** means that the spin of the atoms are mostly randomly oriented; alignment only occurs infrequently in small local areas

1. Intuitively, why 1-dimensional Ising model does not show phase transition between ferromagnetism and paramagnetism? Why 2-dimensional (or 3-dimensional or higher Ising model) shows phase transition?

Mathematically 1D and 2D partition function can be solved with matrix transformations, which reveals a critical temperature Tc =2.27 J / KB  for 2D Ising lattice. Below this critical temperature, the lattice can be easily magnetized, also called very high magnetic susceptibility or permeability. Above this critical temperature, the magnetic susceptibility drops drastically; the lattice becomes paramagnetic.

While in 1D Ising model, it remains paramagnetic no matter how low the temperature is. In other words, there is no long-term order of the lattice (which a line for 1D case). Spins of atoms far away are only very weakly correlated with each other. Ironically, Ising solved this 1D model mathematically, but incorrectly generalized that this model does not exhibit phase behavior in any higher dimension.

Intuitively, for a 1D lattice (a line), each spin only has 2 direct neighbors; the interaction between atoms decays fast with distance. For 2D lattice, each spin has 4 direct neighbors even excluding the diagonal ones, the interactions between spins can propagate farther which eventually bring long-term order if the temperature is low enough (when temperature is low, there is less randomness from thermos fluctuations). Consequently, 3D or higher Ising lattices also shows phase transition at certain critical temperatures.

1. What is partition function? Slide 13

A **partition function** describes the [statistical](https://en.wikipedia.org/wiki/Statistics) properties of a system in [thermodynamic equilibrium](https://en.wikipedia.org/wiki/Thermodynamic_equilibrium). Partition functions are [functions](https://en.wikipedia.org/wiki/Function_(mathematics)) of the thermodynamic [state variables](https://en.wikipedia.org/wiki/State_function), such as the [temperature](https://en.wikipedia.org/wiki/Temperature) and [volume](https://en.wikipedia.org/wiki/Volume). Most of the aggregate [thermodynamic](https://en.wikipedia.org/wiki/Thermodynamics) variables of the system, such as the [total energy](https://en.wikipedia.org/wiki/Energy), [free energy](https://en.wikipedia.org/wiki/Thermodynamic_free_energy), [entropy](https://en.wikipedia.org/wiki/Entropy), and [pressure](https://en.wikipedia.org/wiki/Pressure), can be expressed in terms of the partition function or its [derivatives](https://en.wikipedia.org/wiki/Derivative). The partition function is dimensionless.

It usually takes quite some mathematical efforts to derive/simplify the partition function of a system. But once partition function is derived, most of physical properties of the system can be calculated fairly easily. It took years for Onsager to calculate the exact solution for 2D Ising lattice partition function. Because “he had a lot of time during World War II” (according to himself), as a physics, chemistry professor at Yale Univ.

1. Probability of flip. slide 14

Please note that if Pflip > 1, it’s set to 1. (probability can never be more than 100%)

1. Show can spin value be continuous? Slide 14

We know all spins have discrete values in quantum mechanics. In this research we use spin value in an Ising lattice to represent the water/ice concentration therefore we introduce continuous spin value. This is a practical choice, without strict compliance with quantum mechanics per se. I am happy to discuss more details with you offline on how to improve this model either theoretically or practically.

1. Why do we need this additional term Inertial factor? Slide 14

A standard Ising model with spin +1 and -1 means that each flip will simply change the sign of the cell. However when spin value is continuous, we want to differentiate the probability of flipping to different spin values. Now you can explain more according to the last 2 paragraphs of the script for this slide

1. Why choose simulation period to be half month? Slide 16

See paper

1. Why can we set to 1? Slide 16

See paper

1. Why J is constant across the full lattice? But B is set to be dependent on the location of each spin? Slide 16

See paper. Simply put, J represents binding between water/water or ice/ice, naturally it should not depend on the location of water or ice. B is the external field i.e. can be different ambient temperature, sunlight etc, which can be function of location.

1. Why do we choose B to be linear function of location x and y? slide 16

See paper. our focus area, the north and west side are farther from north pole, close to continent; whereas east/south is very near north pole. Intuitively speaking, we think ambient temperature is colder where closer to pole (south/east), warmer when farther (north/west). Therefore we choose a simple linear function of (x,y) for B and it works pretty well. This choice, of course, can be further explored and enhanced.

1. Metropolis-Hasting algorithm. Slide 17

This is a Markov Chaina Monte Carlo method extremely widely used. (Please try to understand it’s benefits). It obtains a sequence of [random samples](https://en.wikipedia.org/wiki/Pseudo-random_number_sampling) from a [probability distribution](https://en.wikipedia.org/wiki/Probability_distribution) from which direct sampling is difficult. Effectively, from the starting point x, it generates a candidate random nearby point x’, then compare the probability ration r = P(x’)/P(x). If r>=1, it always accepts x’; if r<1 it accepts x’ with probability r. It has been mathematically proven that such a sequence will eventually match the target distribution after sequence that’s long enough. Our simulation follows this algorithm, i.e. we accept spin flip at probability equal to the ratio.

1. What’s dual annealing optimization? Slide 19

See paper. It combine local gradient (fast local optimization) and simulated annealing (probabilistic global optimization). We tried different other packages, like local fast optimization e.g. Newton method, or Nelder-Mead etc., but it does not work well. Dual annealing works best across the methods we tried.

1. Why do we choose 50,000 steps for each period? Slide 19

See paper

1. What does these best-fit parameters mean? Slide 21

See paper. But we focus less on the absolute values of these parameters. As we know, we have chosen Beta=1. If Beta set to different number, all these J, B and I will be scaled accordingly. So absolute value of these parameters do not mean much.

Worth noting that J and I and relatively stable across different time periods which makes sense: why should binding between ice/ice or water/water change much w.r.t time?

But B apparently varies a lot across different time: positive in the melting cycle (June to Sep) and negative in the freezing cycle (Sep to Dec). All these makes perfect sense intuitively

As always, after we find mathematical solution either by solving an equation manually or via computer, we always try to understand them intuitively.

(they might ask whey By shows spikes across period if they look really carefully, then tell them these could be due to irregularities of the ambient environment in this special year 2022; slide 26 for 2012 actually shows slightly different values/trend for By)

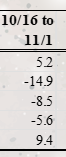
1. Slide 23: on the very left side of the charts, why ice coverage percentage is about 0.9, but ice extent is about 1.0? If extent is 1.0 (100%), should not percentage also be 100%?

Can you explain yourself?

1. How do produce daily results? Slide 25

Read the paper. We simply divide 50,000 steps by 15 to get daily results. Understand it yourself.

the best fit parameter values used in these daily results are from the corresponding period as on slide 21. i.e. daily results on slide 24 is Aug 16 to Sep 1, 2022, so the first column as below are used; slide 25 is Oct 16 to Nov 1st, 2022, so parameters are from the other column below.

1. Slide 28: same question as #16: why left side shows percentage 0.8 but extent 1.0? Are they compatible or bug in code?

Explain it yourself

1. What is quantum ising model?

The **transverse field Ising model** is a quantum version of the classical [Ising model](https://en.wikipedia.org/wiki/Ising_model). It features a lattice with nearest neighbour interactions determined by the alignment of spins along the z axis, and an external field perpendicular to z axis, e.g. in the x axis. An important feature of this setup is that, in a quantum sense, the spin projection along the z axis and along x axis are not commuting oservable quantities. That is, they cannot both be observed simultaneously. This means classical statistical mechanics cannot describe this model, and a quantum treatment is needed. This adds a rich amount of properties to the model. I am just trying to start learning this, but definitely happy to discuss further offline.

More general questions:

1. How does this paper fit into the wider literature of Ising Model application, especially those on environmental study? list 2-3 academic papers that are closely related to or serve as an inspiration for this study?

Ising model has found wide application across various fields as stated in the introduction. My research was inspired by this 2019 paper "Ising model for melt ponds on Arctic sea ice," by Prof Golden of Utah University. Pdf attached. And her is a news link on it: <https://unews.utah.edu/ising-melt-pond/>

Their paper focuses on the equilibrium configuration of melting pond shape and sizes via Ising model but my research studies the dynamic water/ice transition via the kinetic Ising model. There is another paper that simulate dynamic Ising for land pattern (deforestation) transition, as well as a few that have introduced continuous spins to Ising.

1. How does this paper contribute to the literature?

This is the first (I believe it is) Ising model study of the dynamics of sea ice, which not only fits the data very well but also shed light on the importance of environmental factors that can be further investigated in environmental studies and modeling. It can also definitely be further enhanced for deeper and/or wider studies of Ising model application on similar fields

1. What is the limitation of this paper? How can this study be improved?

The model is very simplified. disregarding many real world features e.g. thickness of ice, idiosyncratic variation of external force B across different locations etc. So apparently the results are far from perfect. However it is still able to demonstrate the power of the simple Ising model.

1. Can you think of any other environmental investigations that the Ising Model can be applied to? (relating to question #20)

There are plenty of others, for example, this paper “The kinetic Ising model encapsulates essential dynamics of land pattern change” by Stepinskia and Nowosad studies forest pattern change using classical Ising model.

1. What would be the biggest challenge MODELING WISE if the 2-D model is extended to Quantum Ising?

We only thought about quantum computing, qubits conceptually. Haven’t explored quantum computing in practice. Quantum qubits will be able to represent the continuous spin values, but apparently quantum computer of 3600 qubits is far from practical use yet.

1. What are the environmental factors you can think of that have or have not been incorporated into this study?

Same as question 22

1. What machine (configuration) did you use for simulation? How fast was it ?

All simulations are done on my personal PC. Intel i7 CPU with 12 cores and 64 GB memory; we used parallel processing on CPU; but this research does not even need to use the GeForce GPU on my computer. The computation complexity for Ising model is very reasonable.

* each 50K iteration took less than a second; The time is proportional to iterations, so doubling it to 100k will take twice as much time;
* Dual annealing tried about 10000 set of parameters, taking about 2 hours to complete the whole process of fitting for each simulation period
* There are bounds set for the search (narrow bound for J and I but wider for all Bs). Check the code but don’t have to mention this unless people ask.

1. How did you find this research topic? How much did your mentor help?

My mentor Joanne Wang, a physics Prof at Xiamen University at Malaysia, had been a visiting scholar at Yale University in the past year. During one of our conversations she described this fantastic Ising Model (the legendary Ernst Ising and Lars Onsager, who is a Yale Professor, Nobel Prize winner) ; earlier this year, I came across this Ising ice melting pond paper by Prof Golden and was inspired to use it to model sea ice dynamics. The research turned out to be very interesting and rewarding. I learned a lot of physics in this process.

More technical questions:

1. Why did you choose Metropolis for simulation purpose? Any other paper that also use Metropolis for simulating Ising?

read the paper. there is another Glauber method for Ising simulation which is similar but slightly different from Metropolis. Metropolis is probably the most widely used MCMC method.

1. Is the B function your innovation? Or has it been used in other paper?

read the paper. This is just a practical choice so it works better for the location of our focus area. of course I started with a constant B for the full lattice. but the best fit result was not as great. To accommodate for different external environment for the focus area, a function for B linearly dependent on x and y is a practical choice.

1. What is the biggest challenge of the coding part for this study?

Ising model coding is very straightforward. the most time consuming part is the effort to improve results. For instance, starting without Inertia factor, constant B, then gradually explore these to get better results. These apparently can be further explored and enhanced.

1. What is Bx and what is By? What do they capture? What are the (x, y ) coordinates used?

read the paper